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RF-3612

M. Sc. (Part - II) Examination

April / May - 2010

Mathematics : Paper - 5007

(Special Function)

Time : 3 Hours]

[Total Marks : 70

Instructions :

(1)

નીચે દર્શાવેલ નિશાનીવાળી વિગતો ઉત્તરવહી પર અવશ્ય લખવી.
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Name of the Examination :

Name of the Subject :

Subject Code No. : Section No. (1, 2,.....) :

Seat No. :

Student's Signature

- (2) Attempt all questions
 (3) Notations and conventions are all standard.
 (4) Figure to the right indicates marks.

1 (a) Define uniform convergence of the infinite product. 5

If for a positive constant M_n such that $\sum_{n=1}^{\infty} M_n$ is

convergent and $|a_n(z)| < M_n$ for all z then prove that

the product $\prod_{n=1}^{\infty} (1+a_n)$ is uniformly convergent.

(b) Show that the series $\sum_{n=1}^{\infty} \frac{(a)_n (b)_n}{(c)_n} z^n$ is absolutely 5

convergent for $|z|=1$ if $\text{Re}(c-a-b) > 0$.

- (c) Show that $\prod_{n=1}^{\infty} \left[1 + \frac{1}{(n+1)(n+3)} \right]$ converges and find its value. 4

OR

- 1 (a) Define absolute convergence of an infinite product. 5

Show that the product $\prod_{n=1}^{\infty} (1+a_n)$ with zero factor

electd is absolutely convergent iff $\sum_{n=1}^{\infty} a_n$ is absolutely convergent.

- (b) Obtain the contiguous function relations : 5

$$(\theta + a)F = a F(a+)$$

$$(\theta + c - 1)F = (c - 1) F(c-).$$

- (c) Evaluate $F\left(-\frac{n}{2}, \frac{-n}{2} + \frac{1}{2}; b + \frac{1}{2}; 1\right)$. 4

- 2 (a) If $\left(a + b + \frac{1}{2}\right)$ is neither zero nor a negative integer 5

and if $|x| < 1$ and $|4x(1-x)| < 1$ show that

$$F\left[\begin{matrix} a, b; \\ a + b + \frac{1}{2}; \end{matrix} 4x(1-x)\right] = F\left[\begin{matrix} 2a, 2b; \\ a + b + \frac{1}{2}; \end{matrix} x\right]$$

- (b) Obtain the Neumann polynomials 5

$$O_0(S) = S^{-1}, O_1(S) = S^{-2},$$

$$O_n(S) = O_{n-2}(S) - 2O_{n-1}(S), n \geq 2$$

- (c) If $|z| < 1$ and $\left|\frac{z}{1-z}\right| < 1$ then show that 4

$$F(a, b; c; z) = (1-z)^{-a} F\left(a, c-b, c; \frac{-z}{1-z}\right)$$

OR

- 2 (a) If $(1+a-b)$ is neither zero nor a negative integer, 5
 $\operatorname{Re}(b) < 1$ for convergence then prove that

$$F\left[\begin{matrix} a, b; \\ 1+a-b; \end{matrix} -1 \right] = \frac{\sqrt{(1+a-b)} \left(1 + \frac{a}{2}\right)}{\left(1 + \frac{a}{2} - b\right) \sqrt{(1+a)}}$$

- (b) Define Bessel function. State and prove the Bessel's 5
integral formula.
- (c) Show that $Z J_n'(z) = Z J_{n-1}(z) - n J_n(z)$. 4

- 3 (a) Define Legendre's polynomials $P_n(x)$ obtain the 5
relation $(2n+1)P_n(x) = P_{n+1}'(x) - P_{n-1}'(x)$.

- (b) For $-1 < x < 1$ show that $|P_n(x)| < 1$. 5
- (c) With usual notation prove that : 4

$$P_n(x) = {}_2F_1\left(-n; n+1; 1; \frac{1-x}{2}\right)$$

OR

- 3 (a) Show that $\int_0^1 P_n^2(x) dx = \frac{2}{2n+1}$. 5

- (b) Prove that : 5

$$\sum_{n=0}^{\infty} \frac{P_n(x) t^n}{(n!)^2} = {}_0F_1\left[-; 1; \frac{t}{2}(x+1)\right] {}_0F_1\left[-; 1; \frac{t}{2}(x-1)\right]$$

- (c) Show that : 4

$$\sum_{n=0}^{\infty} \frac{P_n(x) t^n}{n!} = e^{xt} J_0\left(t \sqrt{1-x^2}\right)$$

- 4 (a) State and prove the orthogonal property of the 5
Hermite's polynomials $H_n(x)$.

- (b) Obtain the relations : 5

$$x H_n'(x) = n H_{n-1}'(x) + n H_n(x)$$

$$H_n(x) = 2x H_{n-1}(x) - n H_{n-2}(x)$$

- (c) Show that $H_{2n}(0) = (-1)^2 2^{2n} \left(\frac{1}{2}\right)_n$. 4

OR

- 4 (a) Derive the Rodrigues formula for Hermite's polynomials $H_n(x)$. 5

- (b) Show that : 5

$$\sum_{n=0}^{\infty} \frac{H_n(x) H_n(y) t^n}{n!} = (-4t^2)^{-1/2} \exp \left[y^2 - \frac{(y-2xt)^2}{1-4t^2} \right]$$

- (c) Show that : 4

$$D^s H_n(x) = \frac{2^s n! H_{n-s}(x)}{(n-s)!}, \text{ where } D = \frac{d}{dx}.$$

- 5 (a) Show that : 5

$$L_n^{(\alpha)}(xy) = \sum_{m=0}^n \frac{(1-y)^{n-m} y^m L_m^{(\alpha)}(x) (1+\alpha)_n}{(n-m)! (1+\alpha)_m}$$

- (b) Obtain the relation : 5

$$L_n^{(\alpha)}(x) = L_{n-1}^{(\alpha)}(x) + L_n^{(\alpha-1)}(x)$$

- (c) Show that 4

$$e^t {}_0F_1(-; 1+\alpha; -xt) = \sum_{n=0}^{\infty} \frac{L_n^{(\alpha)}(x)}{(1+\alpha)_n} t^n$$

OR

- 5 (a) Show that : $\int_0^{\infty} e^{-x} x^\alpha \left\{ L_n^{(\alpha)}(x) \right\}^2 dx = \frac{(1+\alpha+n)}{n!}$. 5

- (b) Show that : $L_n^{(\alpha)}(x) = \frac{e^x x^{-\alpha}}{n!} D^n \left[e^{-x} x^{n+\alpha} \right]$. 5

- (c) Show that :

$$\frac{1}{(1-t)^{1+\alpha}} \exp \left(\frac{-xt}{1-t} \right) = \sum_{n=0}^{\infty} L_n^{(\infty)}(x) t^n$$